Government of **Western Australia**
School Curriculum and Standards Authority

MATHEMATICS APPLICATIONS

Calculator-free

ATAR course examination 2017

Marking Key

Marking keys are an explicit statement about what the examining panel expect of candidates when they respond to particular examination items. They help ensure a consistent interpretation of the criteria that guide the awarding of marks.

2017/69875

Section One: Calculator-free **35% (53 Marks)**

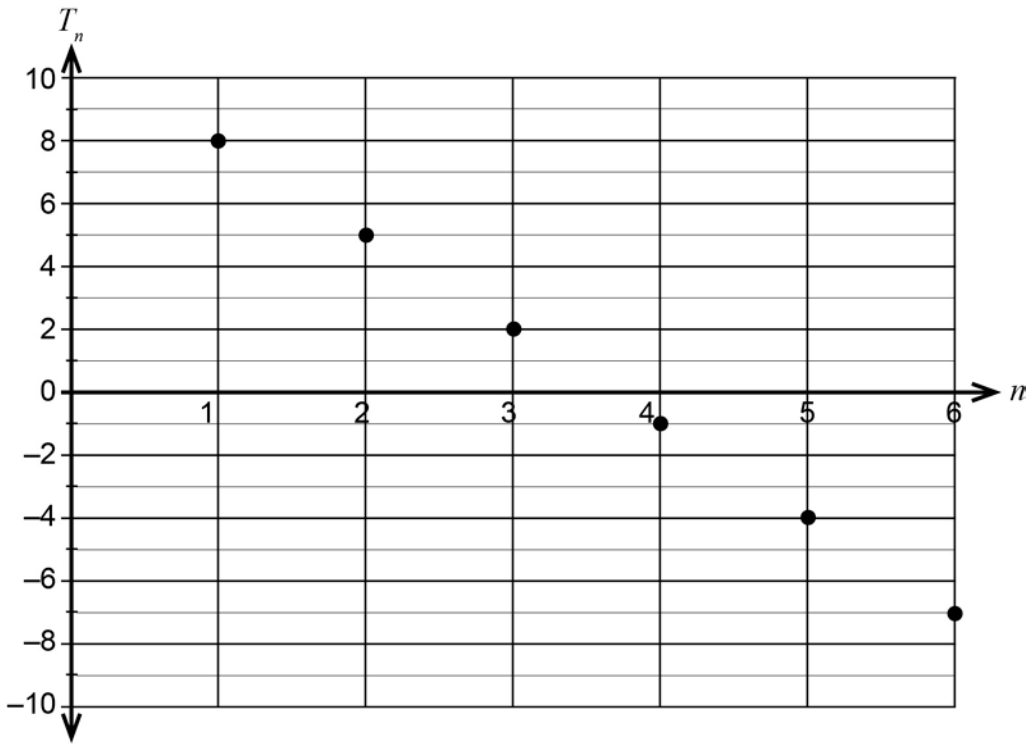
Question 1 **(8 marks)**

Consider the following recurrence relation:

$$T_{n+1} = T_n - 3, \quad T_3 = 2.$$

(a) Display the first six terms of this sequence on the axes below. Label the axes clearly.
(3 marks)

Solution



Specific behaviours

✓ correctly plots at least two terms
✓ correctly labels axes
✓ plots first six terms discretely

(b) (i) Deduce a rule for the n^{th} term of this sequence. (2 marks)

Solution
$T_n = -3n + 11$
Specific behaviours
✓ identifies gradient of –3 and vertical intercept of 11 ✓ uses correct variables, n and T_n

(ii) Hence, determine the first term in the sequence which is less than –500. (3 marks)

Solution
$-500 = -3n + 11$ $3n = 511$ $n = 170.33$ $\therefore 171\text{st term}$ $T_{171} = -3 \times 171 + 11 = -502$
Specific behaviours
✓ solves the equation: $-500 = -3n + 11$ ✓ determines correct term number, i.e. 171st term ✓ determines correct term

Question 2 **(7 marks)**

A supermarket provides a delivery service to its customers. This morning, there are four deliveries (1, 2, 3 and 4) to be made. Each of four drivers, John, Kerry, Liam and Max, is available to do one of the deliveries.

The table below shows the time, in minutes, that each driver would take to complete each of the four deliveries.

Table 1
Delivery Driver

		John	Kerry	Liam	Max
Deliveries	1	35	31	41	36
	2	25	26	33	36
	3	32	28	25	24
	4	27	30	31	28

The store manager will allocate the deliveries so that the total delivery time is at a minimum. He decides to use the Hungarian algorithm to determine the allocation of deliveries to the drivers.

His first step is to subtract the minimum entry in each row from each element, ensuring that each row contains at least one zero.

Table 2
Delivery Driver

		John	Kerry	Liam	Max
Deliveries	1	4	0	10	5
	2	0	1		11
	3	8	4	1	0
	4	0	3	4	1

(a) What is the number missing from the shaded cell? (1 mark)

Solution
8
Specific behaviours
✓ correctly determines missing number

The second step is to ensure that all columns contain at least one zero. The numbers that result from this step are shown in the table below.

Table 3
Delivery Driver

		John	Kerry	Liam	Max
Deliveries	1	4	0	9	5
	2	0	1	7	11
	3	8	4	0	0
	4	0	3	3	1

(b) The smallest number of horizontal and vertical lines that can be drawn to cover all the zeros is three.

(i) Draw in these lines on **Table 3** on the previous page. (1 mark)

Solution

		Delivery Driver			
		John	Kerry	Liam	Max
Deliveries	1	4	0	9	5
	2	0	1	7	11
	3	8	4	0	0
	4	0	3	3	1

Specific behaviours

✓ correctly draws in three lines covering all the zeros

(ii) State why an allocation of delivery drivers cannot be made yet. (1 mark)

Solution

The number of lines (three) is not the same as the number of drivers and tasks (four).

Specific behaviours

✓ correctly states the reason

Question 2 (continued)

(c) Continue the steps of the Hungarian algorithm to determine the optimum allocation of deliveries to the drivers. Complete the table at the bottom of the page and state the minimum total delivery time. (4 marks)

Solution

		Delivery Driver			
		John	Kerry	Liam	Max
Deliveries	1	5	1	9	5
	2	1	2	7	11
	3	10	6	1	1
	4	1	4	3	1

		Delivery Driver			
		John	Kerry	Liam	Max
Deliveries	1	4	0	8	4
	2	0	1	6	10
	3	9	5	0	0
	4	0	3	2	0

Delivery Driver	**John**	**Kerry**	**Liam**	**Max**
Delivery	2	1	3	4

Minimum total delivery time = 109 minutes

Specific behaviours

✓ adds 1 to all covered numbers and 2 to all numbers at intersections
✓ subtracts 1 from every element
OR
✓ adds 1 to all numbers at intersection
✓ subtracts 1 from all uncovered numbers

✓ states correct allocation
✓ states correct minimum total delivery time

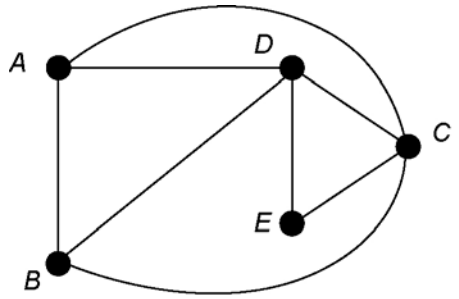
Question 3 **(11 marks)**

(a) A planar graph has five faces and five vertices, A, B, C, D and E.

(i) Determine the number of edges for this graph. (2 marks)

Solution
$5+5-e=2$ $e=8$
Specific behaviours
✓ applies Euler's rule correctly ✓ correctly solves for e

(ii) Draw the planar graph in the space below. (2 marks)

Solution

Specific behaviours
✓ draws graph with correct number of vertices, edges and faces ✓ draws a planar graph

(iii) Determine a Hamiltonian cycle for the graph, giving your answer as a sequence of vertices. (1 mark)

Solution
DECABD There are other possibilities
Specific behaviours
✓ correctly states a Hamiltonian cycle

(iv) Is the graph Eulerian, semi-Eulerian or neither? Justify your answer. (2 marks)

Solution
The graph is semi-Eulerian as it contains exactly two odd vertices
Specific behaviours
✓ correctly states the graph is semi-Eulerian ✓ correctly justifies the answer

Question 3 (continued)

(b) (i) A simple connected graph contains five vertices. Determine the minimum and the maximum number of edges it contains. (2 marks)

Solution
Minimum = 4 Maximum = 10
Specific behaviours
✓ states correct minimum value ✓ states correct maximum value

(ii) A simple connected graph contains n vertices. Determine the minimum number of edges it contains. (1 mark)

Solution
Minimum = $n-1$
Specific behaviours
✓ states correct minimum value

(iii) What name is given to the simple connected graph with the maximum number of edges possible? (1 mark)

Solution
Complete
Specific behaviours
✓ states correct name

Question 4 **(5 marks)**

Ryan was keen to compare interest rates offered by different banks, so he decided to construct a table showing the effective annual rates of interest (%). Part of his table is shown below.

	Rate of interest (p.a.)				
Compounding period	**4%**	**4.5%**	**5%**	**5.5%**	**6%**
Quarterly	4.060	4.577	5.095	5.614	6.136
Monthly	4.074	4.594	5.116	5.641	6.168
Daily	4.081	4.602	5.127	5.654	6.183

(a) Ryan wants to borrow \$5000 to purchase a second-hand car. A bank offers to lend him the money at the rate of 6% p.a. for one year. He plans to pay off the entire loan (including the interest) at the end of the year. Which compounding period should he sign up for? Justify your decision. (2 marks)

Solution
Quarterly As the effective rate of interest is the lowest, he will pay less interest
Specific behaviours
✓ states correct compounding period ✓ states valid reason

(b) Ryan is curious to know how much interest he would earn by investing \$100 for a year, earning 4% p.a. with interest compounded quarterly. Determine the interest he would earn. (1 mark)

Solution
\$4.06
Specific behaviours
✓ states correct amount

(c) Ryan's sister has \$3000 to invest for a year. She has been offered a rate of 5% p.a., with interest compounded daily. Determine the value of her investment at the end of the year. (2 marks)

Solution
$3000 \div 100 = 30$ $30 \times 5.127 = 153.81$ $\therefore \text{Value} = \$3153.81$
Specific behaviours
✓ calculates interest ✓ determines correct value of investment

Question 5 **(9 marks)**

A group of university students was asked the question 'Does full attendance at school lead to an improved examination result?'

The results are summarised below.

	Agree	**Disagree**	**Undecided**
Male under 20 years	8	22	6
Female under 20 years	6	20	8
Male 20 to 25 years	26	7	3
Female 20 to 25 years	30	9	5
Male over 25 years	24	3	2
Female over 25 years	18	2	1

(a) Complete the two-way table below. (2 marks)

	Agree	**Disagree**	**Undecided**
Under 20	14	**42**	**14**
20–25	**56**	**16**	**8**
Over 25	**42**	**5**	3

Solution
See table above
Specific behaviours
✓ completes at least 3 correct entries ✓ completes all correct entries

(b) State the explanatory variable for these data. (1 mark)

Solution
Age
Specific behaviours
✓ states correct variable

(c) The incomplete table below shows row percentages.

	Percentages		
	Agree	**Disagree**	**Undecided**
Under 20	**20**	60	**20**
20–25	**70**	20	**10**
Over 25	84	**10**	**6**

(i) Show how the value of 20% was calculated. (2 marks)

Solution
$\frac{16}{80} \times 100 = 20\%$
Specific behaviours
✓ shows correct numerator ✓ shows correct denominator and calculates the 20%

(ii) Complete the table. (2 marks)

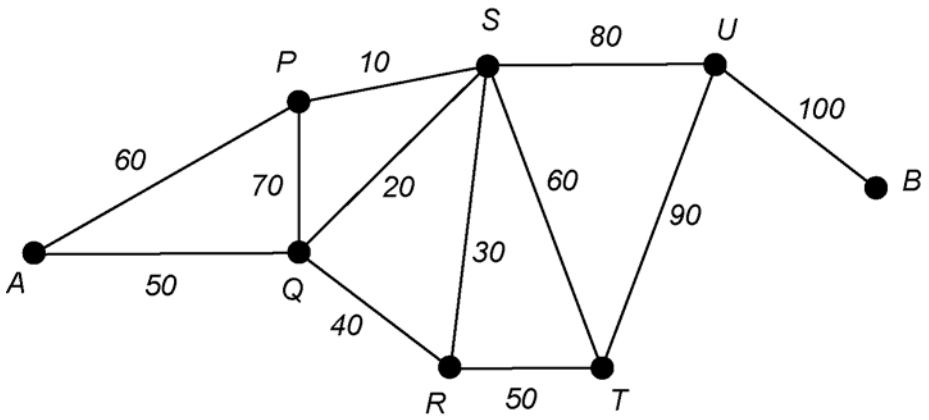
Solution
See above table
Specific behaviours
✓ correctly calculates at least 3 percentages ✓ correctly calculates all percentages

(d) Use the data to determine one association between the variables. Describe the association and explain your reasoning. (2 marks)

Solution
As age increases the percentage of students who agree increases. Percentages in the **Agree** column are increasing with age. There are other possibilities.
Specific behaviours
✓ correctly states an association ✓ gives reasoning

Question 6 **(7 marks)**

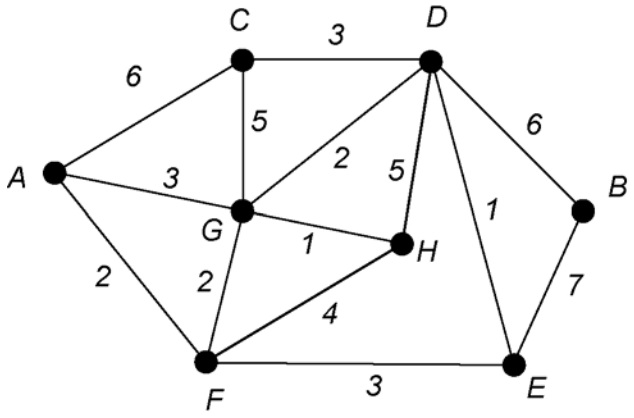
(a) In the network below, the nodes represent towns and the numbers on the arcs represent the time taken (in minutes) to travel between them.



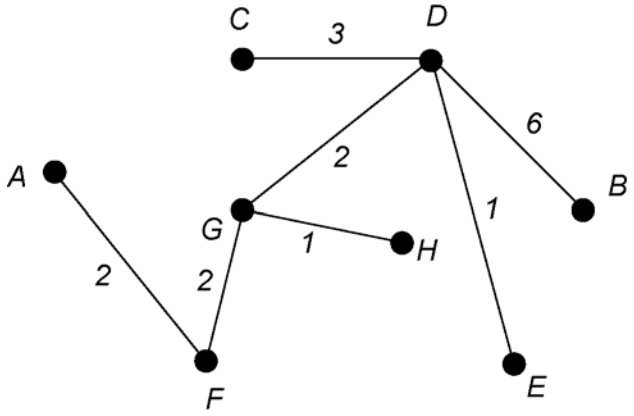
A driver leaves Town A and must deliver goods to all the other towns in the shortest time, finishing at Town B. Determine this shortest time. (A town may be visited more than once). (3 marks)

Solution
AQSPSRTUB Time = 360 minutes
Specific behaviours
✓ lists a route which visits all towns ✓ lists the shortest route ✓ states the shortest time taken

(b) The network below shows the distances (in metres) between stations for a model railway track system.



(i) Determine the minimal spanning tree for the network and draw this tree on the diagram below. (3 marks)



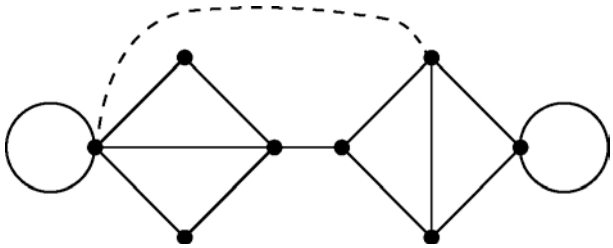
Solution
See graph above
Specific behaviours
✓ draws a tree ✓ draws at least 3 correct edges ✓ draws all correct edges

(ii) State the length of the minimal spanning tree. (1 mark)

Solution
17 metres
Specific behaviours
✓ states correct length

Question 7 **(6 marks)**

(a) The graph below shows the paths connecting the exhibits at a zoo.



(i) Explain why the graph is not semi-Eulerian. (1 mark)

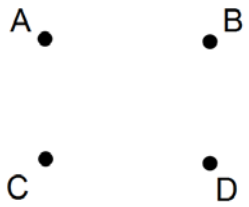
Solution
The graph has more than two odd vertices
Specific behaviours
✓ states correct reason

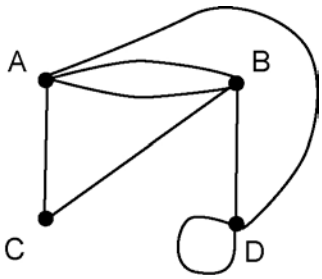
(ii) Draw one edge on the graph so that it becomes semi-Eulerian and does not contain a bridge. (2 marks)

Solution
See above graph (dotted line)
Specific behaviours
✓ draws an edge to make graph semi-Eulerian ✓ draws an edge to make graph semi-Eulerian and does not contain a bridge

(b) The adjacency matrix Q represents the raised paths connecting the observation platforms in the safari section at the zoo. Draw a planar graph for the adjacency matrix. (3 marks)

$$Q = \begin{array}{c} \\ \text{A} \\ \text{B} \\ \text{C} \\ \text{D} \end{array} \begin{array}{c} \begin{array}{cccc} \text{A} & \text{B} & \text{C} & \text{D} \end{array} \\ \begin{bmatrix} 0 & 2 & 1 & 1 \\ 2 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$



Solution

Specific behaviours
✓ draws correct connections between pairs of vertices ✓ draws loop on D ✓ draws a planar graph

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